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Conjugate free convection from long vertical plate fins embedded in a porous medium at high Rayleigh numbers

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Abstract—Analyses are made for steady conjugate free convection about a vertical fin embedded in a porous medium at high Rayleigh numbers. Two types of heat sources at the fin base are considered: a plate heat source and a circular heat source. With the thin-fin approximation for the fin and the boundary-layer approximation for the convective fluid, similarity solutions are obtained for the free convection boundary-layer flow adjacent to a long vertical plate fin with its conductivity–thickness product varying as a power function of distance from the specified origin. Analytical expressions are obtained for the local surface heat flux, local Nusselt number and thermal boundary-layer thickness along the fin. A numerical example, with plots of streamlines and isotherms, is presented for a constant cross-section vertical plate fin heated by a plate heat source and embedded in a porous medium.

INTRODUCTION

In the analysis of heat transfer from a fin, it is customary to assume that the heat transfer coefficient along the fin is a prescribed constant. Experiments have shown that the free convection heat transfer coefficient for a long fin may vary considerably along the fin [1]. The variations of the heat transfer coefficient are due to the interaction between the fin and its adjacent convective flow. Thus, for the analysis of heat transfer from a long fin, the value of the heat transfer coefficient cannot be prescribed *a priori*, and must be determined by a simultaneous consideration of heat conduction in the fin and the convective motion of the adjacent fluid. Such a conjugated free convection problem has been considered by Lock and Gunn [1] who have used the thin-fin approximation for the fin and the boundary-layer approximation for the fluid in their analysis; they show that similarity solutions exist for a class of problems with specific fin geometry. Based on similar approximations and a modified approach, Kuehn *et al.* [2] obtained similarity solutions for the same geometry considered by Lock and Gunn [1]. More general solutions have been obtained numerically by Sparrow and Acharya [3] for free convection about a vertical plate fin with a finite length heated by a plate heat source, and by Tolpadi and Kuehn [4] for a vertical fin with a circular heat source.

The related problems of conjugated free convection from an infinitely long, vertical plate fin embedded in a porous medium and heated by a plate heat source or a circular heat source are considered in this paper. The problem has applications to the design of underground heat exchangers for energy storage and recovery, as well as for temperature control of a catalytic reactor [5]. As in the previous studies [1, 2], the thin-fin approximation for the fin will be invoked and the boundary-layer approximation for the convective fluid outside the fin will be applied. Consistent with these approximations, the effects of curvature and inclination of the fin surface will be neglected. Similarity solutions for free convection about an infinitely long vertical plate fin heated by a plate or circular heat source at its base will be obtained for a class of problems where the fin's conductivity–thickness product varies as a power function of distance from the origin. Numerical computations were carried out for a number of fin shapes. Analytical expressions are obtained for the local surface heat flux, local Nusselt number and thermal boundary-layer thickness along the fin. A numerical example, with plots of streamlines and isotherms, is presented for a constant cross-section vertical plate fin heated by a plate heat source at its base.

LONG VERTICAL PLATE FIN WITH PLATE HEAT SOURCE

Formulation

Consider an infinitely long, vertical plate fin (with thickness $2t_F$) whose base is heated by an isothermal plate heat source embedded in a saturated porous

NOMENCLATURE

c_p specific heat of the convective fluid
f dimensionless similarity stream function defined by equation (12a)
F dimensionless similarity stream function defined by equation (36a)
g acceleration due to gravity
k thermal conductivity of the saturated porous medium
K permeability of the porous medium
n fin shape exponent defined by equations (8) and (32)
Nu local Nusselt number, $Nu = \dfrac{q_F X_b}{k(T_F - T_\infty)}$ for a plate heat source and $Nu = \dfrac{q_F R_b}{k(T_F - T_\infty)}$ for a circular heat source
q_F local fin heat flux
r dimensionless vertical radial coordinate, R/R_b
R vertical radial coordinate
R_b similarity length parameter defined by equation (42)
Ra_{R_b} modified Rayleigh number based on R_b, $\rho_\infty g\beta K(T_b - T_\infty)R_b/(\mu\alpha)$
Ra_{X_b} modified Rayleigh number based on X_b, $\rho_\infty g\beta K(T_b - T_\infty)X_b/(\mu\alpha)$
t_F half-thickness of the vertical fin
T temperature
u dimensionless Darcy velocity in the x- and r-directions, $X_b U/\alpha$, $R_b U/\alpha$
U Darcy velocity in the X- and R-directions
v dimensionless Darcy velocity in the y-direction, $X_b V/\alpha$, $R_b V/\alpha$
V Darcy velocity in the Y-direction
x dimensionless vertical coordinate, X/X_b
X vertical coordinate
X_b similarity length parameter defined by equation (18)
y dimensionless horizontal coordinate, Y/X_b, Y/R_b
Y horizontal coordinate.

Greek symbols
α equivalent thermal diffusivity
β coefficient of thermal expansion
δ_T dimensionless thermal boundary-layer thickness
ε porosity of the porous medium
ζ dimensionless similarity variable defined by equation (36c)
ζ_T value of ζ at the edge of thermal boundary layer
η dimensionless similarity variable defined by equation (12c)
η_T value of η at the edge of thermal boundary layer
θ, Θ similarity temperature, ϕ/ϕ_F
$\hat{\theta}$ angle measured from lower stagnation streamline
λ exponent of the fin temperature
μ viscosity of the convective fluid
ρ density of the convective fluid
ϕ dimensionless fluid temperature, $(T - T_\infty)/(T_b - T_\infty)$
ϕ_F dimensionless fin temperature, $(T_F - T_\infty)/(T_b - T_\infty)$
ψ dimensionless stream function defined by equation (4)
Ψ dimensionless stream function defined in equation (27).

Subscripts
b base of the fin
f convective fluid
F fin
∞ infinity
s unsaturated porous medium.

medium of infinite extent. As shown in Fig. 1, X is the vertical coordinate located along the center line of the fin and Y is the horizontal coordinate perpendicular to the fin. The origin of the coordinates is chosen such that the distance from the origin to the base of the fin is X_b, whose value will be determined later. The prescribed temperature at the base of the fin is T_b, which is higher than that of the surrounding porous medium at T_∞. This results in an upward movement of the convective fluid toward the fin base by the buoyancy force. The mathematical formulation of the problem is unchanged if the fin is cooler than the surrounding porous medium and the fin is inverted.

Under the boundary-layer approximation, the dimensionless governing equations for the convective fluid outside the vertical fin are [6]

$$\frac{\partial^2 \psi}{\partial y^2} = -Ra_{X_b}\frac{\partial \phi}{\partial y} \quad (1)$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = \frac{\partial^2 \phi}{\partial y^2}. \quad (2)$$

Here $x = X/X_b$, $y = Y/X_b$ and $Ra_{X_b} = \rho_\infty g\beta K(T_b - T_\infty)X_b/\mu\alpha$ where ρ, μ and β are the density, viscosity, and the thermal expansion coefficient of the fluid; K is the permeability or the porous medium; g is the gravitational acceleration; $\alpha = k/(\rho_\infty c_p)_f$ is the equivalent thermal diffusivity with $(\rho_\infty c_p)_f$ denoting the heat

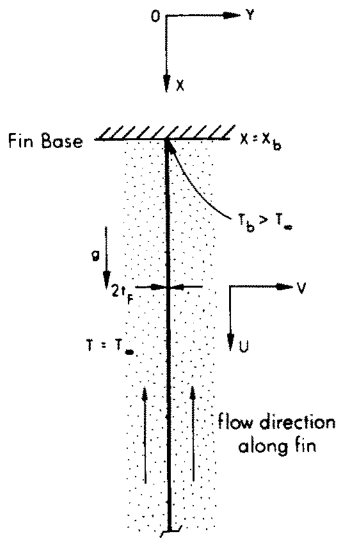

FIG. 1. Vertical plate fin with plate heat source.

capacity of the fluid where c_p is the specific heat of the fluid, and k is the stagnant thermal conductivity of the saturated porous medium given by $k = (1-\varepsilon)k_s + \varepsilon k_f$ with ε denoting the porosity of the medium and k_s and k_f denoting the thermal conductivity of the solid and the convecting fluid, respectively. The dependent variables in equations (1) and (2) are the dimensionless temperature and the dimensionless stream functions which are defined as

$$\phi = \frac{T - T_\infty}{T_b - T_\infty} \tag{3}$$

and

$$u = \frac{UX_b}{\alpha} = \frac{\partial\psi}{\partial y} \quad \text{and} \quad v = \frac{VX_b}{\alpha} = -\frac{\partial\psi}{\partial x} \tag{4a,b}$$

where U and V are the Darcian velocities in the X- and Y-directions while u and v are the corresponding dimensionless quantities.

The boundary conditions for the fluid at infinity are

$$y \to \infty: \quad \frac{\partial\psi}{\partial y} = 0 \quad \text{and} \quad \phi = 0 \tag{5a,b}$$

while those at the fin–fluid interface are

$$y = 0: \quad \frac{\partial\psi}{\partial x} = 0 \quad \text{and} \quad \phi = \phi_F \tag{6a,b}$$

where $\phi_F = (T_F - T_\infty)/(T_b - T_\infty)$ with T_F denoting the temperature along the fin which is to be determined. Note that the thin-fin approximation is used in equation (6) such that the thickness of the fin can be neglected.

Under the thin-fin approximation, there is no temperature variation across the fin and heat conduction in the fin is quasi-one-dimensional. It follows that the governing equation for heat conduction in a thin fin of variable conductivity or thickness is given by

$$\frac{d}{dx}\left(k_F \frac{t_F}{X_b}\frac{d\phi_F}{dx}\right) + k\left(\frac{\partial\phi}{\partial y}\right)_{y=0} = 0 \tag{7}$$

where k_F and t_F are the thermal conductivity and half-thickness of the fin. The conductivity–thickness product will now be assumed to be of the form

$$k_F t_F = k_b t_b x^n \tag{8}$$

where n is the prescribed fin shape exponent and the subscript b refers to the condition at the fin base.

The boundary conditions for equation (7) are

$$x = 1, \quad \phi_F = 1 \tag{9}$$

$$x \to \infty, \quad \phi_F = 0. \tag{10}$$

Similarity solution

From the work of Cheng and Minkowycz [7], it is shown that a similarity solution exists for the convecting fluid if

$$\phi_F = x^\lambda \tag{11}$$

and that the similarity solution is of the form

$$\psi = \sqrt{Ra_{X_b}}\, x^{(\lambda+1)/2} f(\eta) \tag{12a}$$

$$\theta(\eta) = \frac{\phi}{\phi_F} \tag{12b}$$

and

$$\eta = \sqrt{Ra_{X_b}}\, x^{(\lambda-1)/2} y \tag{12c}$$

where f and θ are determined from

$$f'' = -\theta' \tag{13}$$

$$\theta'' = \lambda f'\theta - \left(\frac{\lambda+1}{2}\right) f\theta' \tag{14}$$

subject to the boundary conditions

$$f(0) = \theta(0) - 1 = 0 \tag{15a,b}$$

$$f'(\infty) = \theta(\infty) = 0 \tag{16a,b}$$

where the primes indicate differentiation with respect to η. Integrating equation (13) once and with the aid of equations (16) gives

$$f' = -\theta. \tag{17}$$

Substituting equations (11) and (12) into (7) gives

$$X_b = \left\{ \frac{k_b t_b (2n-3)(3n-4)}{k\sqrt{\dfrac{\rho_\infty g\beta K(T_b - T_\infty)}{\mu\alpha}}\,[-\theta'(0)]} \right\}^{2/3} \tag{18}$$

provided that

$$\lambda = 2n - 3. \tag{19}$$

Note that equation (11) satisfies the boundary conditions (9) and (10) if

$$\lambda < 0. \tag{20}$$

Substituting equation (19) into (20) yields

$$n < \tfrac{3}{2}. \tag{21}$$

With the aid of equation (19), equation (14) becomes

$$\theta'' = (2n-3)f'\theta-(n-1)f\theta'. \tag{22}$$

Equations (11) and (12) in terms of the fin shape exponent n become

$$\phi_F = x^{2n-3} \tag{23}$$

$$\psi = \sqrt{Ra_{X_b}}\,x^{n-1}f(\eta) \tag{24a}$$

$$\phi = x^{2n-3}\theta(\eta) \tag{24b}$$

$$\eta = \sqrt{Ra_{X_b}}\,x^{n-2}y. \tag{24c}$$

Equations (23) and (24) with f and θ determined from equations (17) and (22) subject to the boundary conditions (15) and (16b) are the solution for the problem of conjugate free convection about a plate fin heated by a plate heat source.

LARGE VERTICAL PLATE FIN WITH CIRCULAR HEAT SOURCE

Formulation

Consider next an infinitely large, vertical plate fin with thickness $2t_F$ heated by an isothermal circular heat source at its base. The plate fin is embedded in a saturated porous medium as shown in Fig. 2, where $\hat{\theta}$ is the angle measured from the lower stagnation streamline, R is the radial coordinate near the lower symmetry line of the fin ($\hat{\theta} \sim 0$) and Y is the horizontal coordinate perpendicular to the fin. The origin of the coordinate system is at the center of the circular heat source. As before, the prescribed temperature at the base of the fin is T_b, which is higher than that of the surrounding porous medium at T_∞.

Under the boundary-layer approximation, the dimensionless governing equations for the convecting fluid near the lower stagnation region are

$$\frac{1}{r}\frac{\partial^2\Psi}{\partial y^2} = -Ra_{R_b}\frac{\partial\Phi}{\partial y} \tag{25}$$

$$\frac{1}{r}\left(\frac{\partial\Psi}{\partial y}\frac{\partial\Phi}{\partial r}-\frac{\partial\Psi}{\partial r}\frac{\partial\Phi}{\partial y}\right) = \frac{\partial^2\Phi}{\partial y^2} \tag{26}$$

where $r = R/R_b$, $y = Y/R_b$ and $Ra_{R_b} = \rho_\infty g\beta K(T_b - T_\infty)R_b/\mu\alpha$ with R_b denoting the radius of the circular heat source. The dimensionless stream function is defined as

$$ru = \frac{\partial\Psi}{\partial y}, \quad rv = -\frac{\partial\Psi}{\partial r} \tag{27a,b}$$

while the dimensionless temperature is defined by

$$\Phi = \frac{T-T_\infty}{T_b-T_\infty}. \tag{28}$$

The boundary conditions for the fluid outside the fin are

$$y\to\infty: \quad \frac{\partial\Psi}{\partial y} = 0 \quad \text{and} \quad \Phi = 0 \tag{29a,b}$$

and

$$y = 0: \quad \frac{\partial\Psi}{\partial r} = 0 \quad \text{and} \quad \Phi = \Phi_F. \tag{30a,b}$$

Under the thin-fin approximation, the governing equation for heat conduction in the fin is given by

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{k_F t_F}{R_b}\frac{d\Phi_F}{dr}\right]+k\left(\frac{\partial\Phi}{\partial y}\right)_{y=0} = 0 \tag{31}$$

where the prescribed conductivity–thickness product is assumed to be of the form

$$k_F t_F = k_b t_b r^n. \tag{32}$$

Equation (32) is subject to the boundary conditions

$$r = 1: \quad \Phi_F = 1 \tag{33}$$

$$r\to\infty: \quad \Phi_F = 0. \tag{34}$$

Similarity solution

It can be shown that a similarity solution exists for the fluid outside the fin if

$$\Phi_F = r^\lambda \tag{35}$$

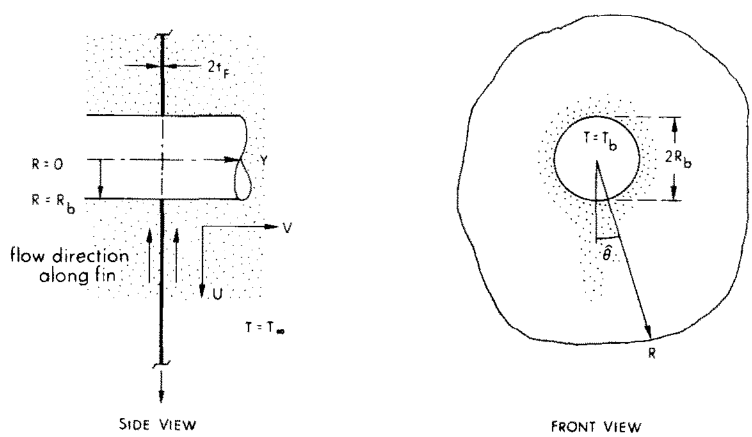

FIG. 2. Vertical plate fin with circular heat source.

and that the similarity solution is of the form

$$\Psi = Ra_{R_b} r^{(\lambda+3)/2} F(\zeta) \quad (36a)$$

$$\Theta(\zeta) = \frac{\Phi}{\Phi_F} \quad (36b)$$

$$\zeta = \sqrt{Ra_{R_b}}\, r^{(\lambda-1)/2} y \quad (36c)$$

where F and Θ are determined from

$$F'' = -\Theta' \quad (37)$$

$$\Theta'' = \lambda F'\Theta - \left(\frac{\lambda+3}{2}\right) F\Theta' \quad (38)$$

subject to the boundary conditions

$$F(0) = \Theta(0) - 1 = 0 \quad (39a,b)$$

$$F'(\infty) = \Theta(\infty) = 0 \quad (40a,b)$$

where the primes indicate differentiation with respect to ζ. Integrating equation (37) once and with the aid of (40a,b) gives

$$F' = -\Theta. \quad (41)$$

Substituting equations (35) and (36b) into (31) yields

$$R_b = \left\{ \frac{3k_b t_b (2n-3)(n-1)}{k\sqrt{\dfrac{\rho_\infty g \beta K (T_b - T_\infty)}{\mu\alpha}}\,[-\Theta'(0)]} \right\}^{2/3} \quad (42)$$

provided that

$$\lambda = 2n - 3. \quad (43)$$

Note that equation (35) satisfies boundary conditions (33) and (34) if

$$\lambda < 0. \quad (44)$$

Substituting equation (43) into equation (44) gives

$$n < \tfrac{3}{2}. \quad (45)$$

Equations (43)–(45) are identical to equations (19)–(21) for the case of a plate heat source. With the aid of equation (43), (38) becomes

$$\Theta'' = (2n-3)F'\Theta - nF\Theta'. \quad (46)$$

Equations (35) and (36) in terms of the fin shape exponent n become

$$\Phi_F = r^{2n-3} \quad (47)$$

$$\Psi = \sqrt{Ra_{R_b}}\, r^n F(\zeta) \quad (48a)$$

$$\Phi = r^{2n-3}\Theta(\zeta) \quad (48b)$$

$$\zeta = \sqrt{Ra_{R_b}}\, r^{n-2} y \quad (48c)$$

where F and Θ are determined from equations (41) and (46) subject to the boundary conditions (39) and (40b).

RESULTS AND DISCUSSION

Equations (21) and (45) show that solutions exist for the value of the fin shape exponent less than 3/2. Note that for a fin with constant thermal conductivity $n < 0$ for a tapered fin. For this reason, computations were carried out for the following values of the fin shape exponent: $n = 0, -1/2, -1$ and -2. Results for f and F are shown in Fig. 3, while those of θ and Θ are shown in Fig. 4. In these figures, the solid lines are for the plate heat source while the dashed lines are for the circular heat source. It is shown that the magnitude of these functions decrease as n is decreased from zero. The variations of $-\theta'(\eta)$ and $-\Theta'(\zeta)$ at two values of n are presented in Fig. 5 which shows that the effect of n depends on the value of η and ζ.

It follows from equations (4), (12a,c), (27) and (36a,c) that

$$u = \begin{cases} -(Ra_{X_b})x^{2n-3}\theta(\eta), & \text{for a plate heat source} \quad (49a) \\ -(Ra_{R_b})r^{2n-3}\Theta(\zeta), & \text{for a circular heat source} \quad (49b) \end{cases}$$

$$v = \begin{cases} -\sqrt{Ra_{X_b}}\, x^{n-2}[(n-1)f + (n-2)\eta f'], & \text{for a plate heat source} \quad (50a) \\ -\sqrt{Ra_{R_b}}\, r^{n-2}[nF + (n-2)\zeta F'], & \text{for a circular heat source.} \quad (50b) \end{cases}$$

Equations (49) show that the convective flow is moving vertically upward. Under the constraints of equations (21) and (45), we note that $2n-3 < 0$. It follows from

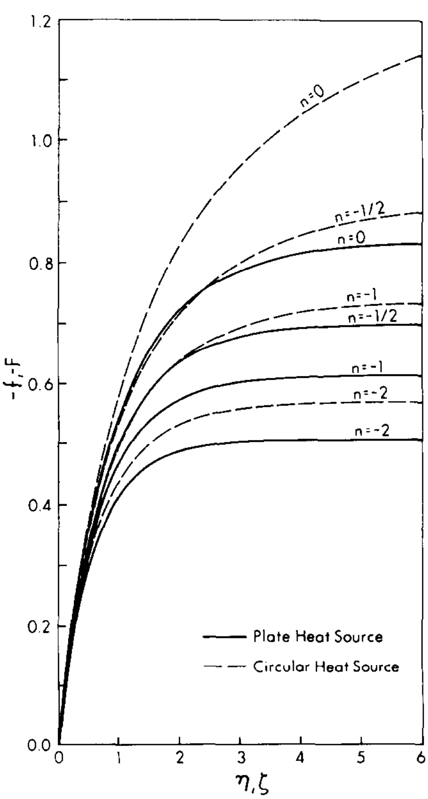

FIG. 3. The effect of fin shape exponents on dimensionless stream functions for plate and circular heat source fins.

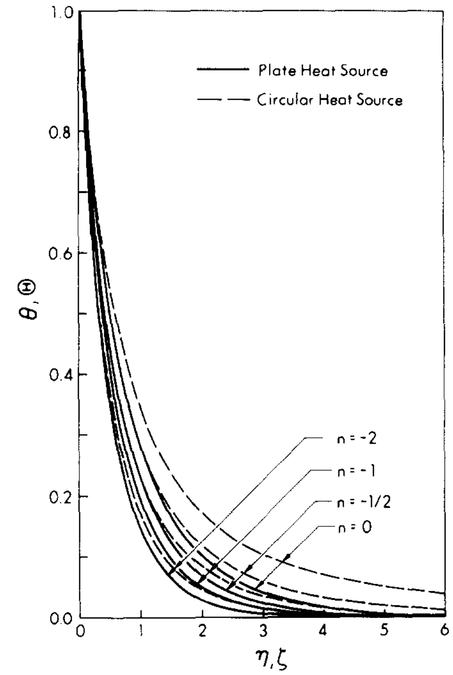

FIG. 4. Dimensionless temperature profiles for various values of the fin shape exponent for fins with plate and circular heat sources.

(49) that the vertical velocity increases from zero at the tip of the fin and accelerates toward the base, i.e. toward the origin of the coordinate system.

The local surface heat flux along the fin is given by

$$q_F = -k\left(\frac{\partial T}{\partial Y}\right)_{Y=0}$$

$$= \begin{cases} k\sqrt{Ra_{X_b}}\dfrac{(T_b - T_\infty)}{X_b}x^{3n-5}[-\theta'(0)] & \text{for a plate heat source} \quad (51a) \\ k\sqrt{Ra_{R_b}}\dfrac{(T_b - T_\infty)}{R_b}r^{3n-5}[-\Theta'(0)], & \text{for a circular heat source} \quad (51b) \end{cases}$$

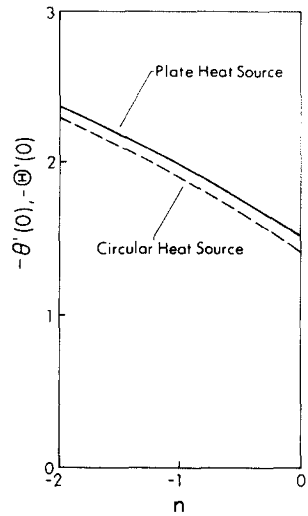

FIG. 6. Dimensionless surface temperature gradients as a function of the fin shape exponent for fins with plate and circular heat sources.

where the values of $-\theta'(0)$ and $-\Theta'(0)$ as a function of n are presented in Fig. 6 and listed in Table 1. It is shown in Fig. 6 that for a given value of n, the value of $-\theta'(0)$ is higher than that of $-\Theta'(0)$. Equations (51) can be expressed in the following dimensionless form

$$Nu = \begin{cases} \sqrt{Ra_{X_b}}\,x^{n-2}[-\theta'(0)], & \text{for a plate heat source} \quad (52a) \\ \sqrt{Ra_{R_b}}\,r^{n-2}[-\Theta'(0)], & \text{for a circular heat source} \quad (52b) \end{cases}$$

where Nu is the local Nusselt number which is defined as

$$Nu = \frac{q_F X_b}{k(T_F - T_\infty)}$$

for a plate heat source and

$$Nu = \frac{q_F R_b}{k(T_F - T_\infty)}$$

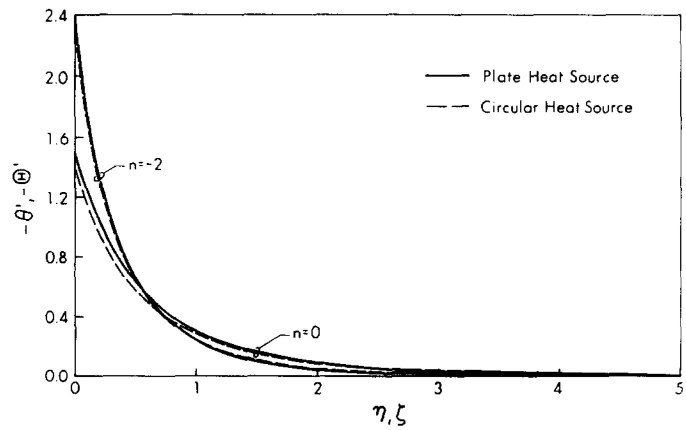

FIG. 5. The variations of dimensionless temperature gradients for fins with plate and circular heat sources at two values of the fin shape exponent.

Table 1. Values of $-\theta'(0)$, η_T, $-\Theta'(0)$ and ζ_T for selected values of n

n	$-\theta'(0)$	η_T	$-\Theta'(0)$	ζ_T
0.00	1.516	4.64	1.414	12.72
−0.25	1.646	4.15	1.500	8.18
−0.50	1.766	3.79	1.675	6.27
−0.75	1.878	3.51	1.792	5.24
−1.00	1.985	3.28	1.902	4.58
−1.50	2.182	2.94	2.106	3.77
−2.00	2.362	2.68	2.292	3.27

for a circular heat source. Under the constraints of equations (21) and (45), we note that $3n-5 < 0$ and $n-2 < 0$. It follows from equations (51) and (52) that both q_F and Nu increase from zero at the tip of the fin toward the base.

Finally, consider the thermal boundary-layer thickness adjacent to the fin. If we define $y = \delta_T$ as the edge of the thermal boundary layer where θ or Θ has a value of 0.01, and let η_T or ζ_T be the value of η or ζ at this point, it follows from (12c) and (48c) that

$$\delta_T = \begin{cases} \dfrac{\eta_T}{\sqrt{Ra_{x_b}}\, x^{n-2}}, & \text{for a plate heat source} \quad (53a) \\ \dfrac{\zeta_T}{\sqrt{Ra_{R_b}}\, r^{n-2}}, & \text{for a circular heat source} \quad (53b) \end{cases}$$

where the values of η_T and ζ_T as a function of n are listed

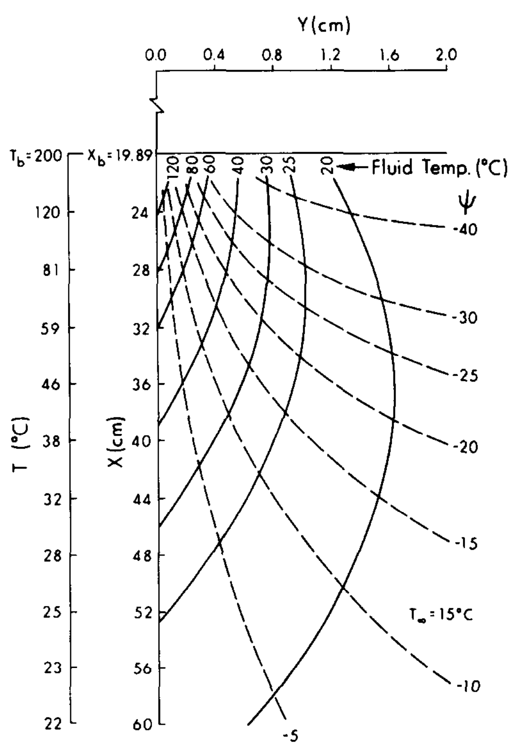

FIG. 7. Streamlines and isotherms near a copper plate heat source fin embedded in a geothermal reservoir.

in Table 1. Equations (53) show that the thermal boundary-layer thickness increases from zero at the tip of the fin toward the base for $n < 0$.

To provide a sense of the magnitude of the physical quantities involved, computations were carried out for a long copper fin with constant thermal conductivity ($k_b = 90\,\text{cal m}^{-1}\,\text{s}^{-1}\,°\text{C}^{-1}$) and with constant thickness ($2t_b = 0.01$ m). The fin is heated by a plate heat source at 200°C, and is embedded in a geothermal reservoir with $K = 10^{-8}$ m^2, $k = 0.58$ cal m^{-1} s^{-1} °C^{-1} at $T_\infty = 15$°C. Other physical properties used in the computations are $\rho_\infty = 10^6$ g m^{-3}, $g = 9.8$ m s^{-2}, $\beta = 1.8 \times 10^{-4}$ °C^{-1}, $\mu = 0.27$ g m^{-1} s^{-1}, $\alpha = 6.3 \times 10^{-7}$ m^2 s^{-1}. Results of the computations for streamlines (dashed lines) and isotherms (solid lines) are presented in Fig. 7. It is seen that some of the isotherms curve back toward the fin as they approach the plate heat source. The reason for this behavior is the streamwise increase of the temperature of the fin which causes a streamwise increase in the buoyancy force, resulting in an additional acceleration of the flow.

CONCLUDING REMARKS

In this paper, similarity solutions have been obtained for the problem of conjugate free convection of a Darcian fluid about long vertical plate fins with a power law variation of conductivity–thickness product. Both a plate heat source and a circular heat source have been considered. For the similarity solution to exist, the length scale of these two problems must satisfy equation (18) for a plate heat source and equation (42) for a circular heat source. Since the origin of the coordinate system for the case of a plate heat source is not specified *a priori*, equation (18) can be used to locate the origin of the coordinate system for a particular fin under consideration. Thus, the similarity solution obtained for the case of a plate heat source is quite general. For the case of the circular heat source, since the radius of the circular heat source R_b and the origin of the coordinate are prescribed for a particular fin, equation (42) places a restriction on the condition for which a similarity solution exists. A study of more general nature, involving *nonsimilar* analysis of conjugate heat transfer in porous media, is presently being done by Liu and Minkowycz [8].

REFERENCES

1. G. S. Lock and J. C. Gunn, Laminar free convection from a downward-projecting fin, *J. Heat Transfer* **90**, 63–70 (1968).
2. T. H. Kuehn, S. S. Kwon and A. K. Tolpadi, Similarity solution for conjugate natural convection heat transfer from a long vertical fin, *Int. J. Heat Mass Transfer* **26**, 1718–1721 (1983).
3. E. M. Sparrow and S. Acharya, A natural convection fin with a solution-determined nonmonotonically varying heat transfer coefficient, *J. Heat Transfer* **103**, 218–225 (1981).
4. A. K. Tolpadi and T. H. Kuehn, Conjugate three-dimensional natural convection heat transfer from a

horizontal cylinder with long transverse plate fins, *Num. Heat Transfer* **7,** 319–342 (1984).
5. W. Adam and D. Vortmeyer, Effect of conducting inserts on ignition and extinction behavior of a fixed bed catalytic reactor, *Germ. chem. Engng* **3,** 1192–1200 (1980).
6. P. Cheng, Heat transfer in geothermal systems, *Adv. Heat Transfer* **14,** 1–105 (1978).
7. P. Cheng and W. J. Minkowycz, Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike, *J. geophys. Res.* **82,** 2040–2044 (1977).
8. J.-Y. Liu, Conjugate mixed convection–conduction heat transfer in porous media. Ph.D. thesis, University of Illinois at Chicago (1985).

CONVECTION NATURELLE CONJUGUEE SUR DE LONGUES AILETTES VERTICALES ET PLANES, NOYEES DANS UN MILIEU POREUX, AUX NOMBRES DE RAYLEIGH ELEVES

Résumé—On analyse la convection naturelle permanente sur une ailette verticale dans un milieu poreux aux nombres de Rayleigh élevés. Deux types de sources de chaleur sont considérés à la base de l'ailette : une source plane et une source circulaire. Avec l'approximation de l'ailette mince et celle de la couche limite, des solutions en similitude sont obtenues pour l'écoulement de convection naturelle adjacent à une ailette verticale plane et longue avec son produit conductivité–épaisseur variant comme une fonction puissance de la distance à une origine spécifiée. Des expressions analytiques sont obtenues pour le flux thermique pariétal local, le nombre de Nusselt local et l'épaisseur de couche limite thermique le long de l'ailette. Un exemple numérique, avec graphes de lignes de courant et d'isothermes, est présenté pour une ailette plane verticale à section droite constante, chauffée par une source de chaleur plane et noyée dans un milieu poreux.

FREIE KONVEKTION AN LANGEN VERTIKALEN, IN EIN PORÖSES MEDIUM EINGEBETTETEN PLATTENRIPPEN BEI HOHEN RAYLEIGH-ZAHLEN

Zusammenfassung—Es wurden Untersuchungen für die stationäre freie Konvektion an einer vertikalen, in einem porösen Medium eingebetteten Rippe bei hohen Rayleigh-Zahlen durchgeführt. Zwei Arten von Wärmequellen wurden am Rippenfuß angenommen : eine ebene und eine kreisförmige Wärmequelle. Mit der vereinfachten Betrachtung als dünne Rippe und der Grenzschichtnäherung für das Fluid werden für die Grenzschichtströmung bei freier Konvektion an einer langen vertikalen Plattenrippe Ähnlichkeitslösungen ermittelt. Dabei wurde das Produkt aus Wärmeleitfähigkeit und Rippendicke variiert als Potenzfunktion des Abstandes vom festgelegten Koordinatenursprung. Es wurden analytische Ausdrücke für die lokale Wärmestromdichte an der Oberfläche, für die lokale Nusselt-Zahl und für die thermische Grenzschichtdicke entlang der Rippe gefunden. Es wird ein numerisches Beispiel mit den Plots der Stromlinien und Isothermen in einem Schnitt durch eine vertikale Plattenrippe bei ebener Wärmequelle vorgestellt.

СОПРЯЖЕННАЯ СВОБОДНАЯ КОНВЕКЦИЯ ПРИ БОЛЬШИХ ЧИСЛАХ РЭЛЕЯ ОТ ДЛИННЫХ ВЕРТИКАЛЬНЫХ ПЛОСКИХ РЕБЕР, ПОГРУЖЕННЫХ В ПОРИСТУЮ СРЕДУ

Аннотация—Изучена стационарная сопряженная свободная конвекция при больших числах Рэлея у вертикального ребра, погруженного в пористую среду. Рассмотрены два вида тепловых источников в основании ребра: плоский и круговой. В приближении тонкого ребра и пограничного слоя получены автомодельные решения для свободноконвективного течения пограничного слоя, примыкающего к длинному вертикальному плоскому ребру, когда произведение толщины ребра на теплопроводность представляет степенную функцию расстояния от заданного начала координат. Получены аналитические выражения для локального поверхностного теплового потока, локального числа Нуссельта и толщины теплового пограничного слоя вдоль ребра. Приводится числовой пример с графиками линий тока и изотермами для вертикального плоского ребра постоянного поперечного сечения, нагреваемого плоским тепловым источником и погруженного в пористую среду.